TRANSVERSE SELF-DIFFUSION IN AN ELECTRODYNAMICALLY FLUIDIZED SYSTEM

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The results of statistical modeling of transverse self-diffusion in an electrodynamically fluidized (EF) system of microparticles are presented. The stability of the EF system in the case when the concentration and electric field are nonuniform is discussed.

In [1] the charge distribution function of microparticles is calculated by means of statistical modeling of electrodynamic fluidization of the microparticles, taking into account recombination and the conditions under which the EF system remains stable as the concentration of microparticles increases, are studied. It is shown that the maximum achievable concentration is determined by the critical concentration of microparticles increases, are studied. It is shown that the maximum achievable concentration is determined by the critical concentration of microparticles for which the potential energy of the EF system is maximum. For higher concentrations the EF system is unstable and decomposes into a layer of deposits on the surface of the electrode and a region of electrodynamically fluidized microparticles.

In this paper we present the results of a calculation of the concentration dependence of the self-diffusion coefficient of the EF system, which determines the diffusion flux of microparticles in a direction transverse to the electric field, and the conditions under which the EF system is stable with respect to nonuniformity of the concentration and of the electric field are examined.

Since the phenomenon of self-diffusion is connected with the interaction of particles in the EF system, the calculation of the particle flux, caused by the nonuniformity of the particle concentration, must be based on taking into account interparticle collisions. In [2] the static spatial stability of an EF system was studied and the free path length of a microparticle after a collision with another microparticle was estimated. This estimate, however, is applicable only in the case of low concentration of microparticles, when rare collisions do not lead to neutralization of their charge. For this reason, it is necessary to study the effect of the recombination mechanism on self-diffusion of microparticles for any concentration. In accordance with the model adopted in [1] for the EF system the motion of a microparticle parallel to the electric field is uniquely determined by the charge of the particle: $V = (qE + mg)/s$, $V \parallel E$. For this reason, we shall study self-diffusion of microparticles only in a direction transverse to E. Then for the system of coordinates $x = (x_1, x_2, x_3)$ with $x_3 \parallel E$ we obtain for the diffusion flux, according to [3],

$$J_{D_i} = n\frac{\overline{\Delta x_i}}{t} - \frac{\partial n}{\partial x_i}\frac{\overline{\Delta x_i^2}}{2t} - n\frac{\partial n}{\partial x_i}\frac{\partial}{\partial n}\left(\frac{\overline{\Delta x_i^2}}{2t}\right) + \frac{1}{2}\frac{\partial^2 n}{\partial x_i^2}\frac{\overline{\Delta x_i^3}}{3t} + \left(\frac{\partial n}{\partial x_i}\right)^2\frac{\partial}{\partial n}\left(\frac{\overline{\Delta x_i^3}}{3t}\right) + \dots, \tag{1}$$

where $\overline{\Delta x_i}$, $i = 1, 2$ is the average displacement of a microparticle over the time t. Retaining terms of second order and taking into account the fact that the odd moments of the displacement are equal to zero, we obtain

$$J_{D_i} = -\frac{\partial n}{\partial x_i}\frac{\overline{\Delta x_i^2}}{2t} - n\frac{\partial n}{\partial x_i}\frac{\partial}{\partial n}\left(\frac{\overline{\Delta x_i^2}}{2t}\right) = -\frac{\partial (D_\perp n)}{\partial n}\frac{\partial n}{\partial x_i}, \tag{2}$$

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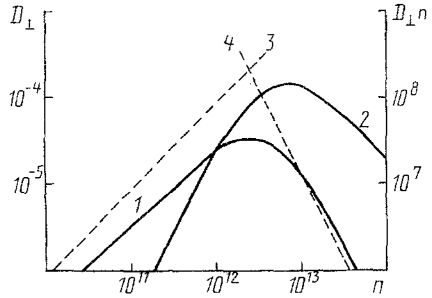

Fig. 1. The self-diffusion coefficient $D_\perp$ (1) and the quantity $D_\perp n$ (2) versus the microparticle concentration n and estimate of $D_\perp^*$ (3) for low concentration, and $D_\perp^{**}$ (4) for high concentration. $D_\perp$, m^2/sec; $D_\perp n$, 1/(msec); n, 1/m^3.

where $D_\perp = \overline{\Delta x_i^2}/2t$ is the transverse self-diffusion coefficient. In the case of $D_\perp$, which is independent of the concentration of microparticles, the expression (2) is Fick's first law.

In statistical modeling, in order to calculate the displacement for each collision of a microparticle with charge q_i and velocity V_i with a microparticle having charge q_j and velocity V_j, the transverse component of the velocity of the ith particle after a collision, according to [2], was assumed to equal

$$V_\perp = |V_i - V_j| \sin\varphi \cos\varphi, \tag{3}$$

where φ is the contact angle between the microparticles, determined by the random impact parameter b, $0 \le b \le 2r$:

$$\sin\varphi = b/2r; \quad \cos\varphi = \sqrt{1-(b/2r)^2}. \tag{4}$$

Under the adopted condition $\tau_r/\tau \ll 1$, where $\tau_r = m/s$ is the relaxation time and τ is the time between collisions, the transverse path length of a microparticle after a collision is equal to $\Delta l = V_\perp \tau_r$. Correspondingly, for $\Delta x_i^2 = \Delta l^2/2$, the self-diffusion coefficient is determined by the expression

$$D_\perp = \frac{\sum\limits_k \Delta l_k^2}{4\sum\limits_k \tau_k}, \tag{5}$$

where τ_k and Δl_k are the free flight time and the transverse displacement of a microparticle after the kth collision.

Figure 1 shows the concentration dependence of $D_\perp$, calculated for microparticles with $r = 2\cdot10^{-6}$ m and $\rho = 7.8\cdot10^3$ kg/m^3 for $E = 2\cdot10^4$ V and $d = 10^{-2}$ m. It is obvious that the self-diffusion coefficient depends strongly on the concentration of the microparticles. For low concentration $D_\perp \sim n$ the self-diffusion coefficient decreases as n increases. The maximum value of $D_\perp$ is observed at $n = n_{\text{init}} = 1/(d\beta)$ — the initial concentration, characterizing the situation in which the collisions of the microparticles play a significant role in the behavior of the microparticles. Correspondingly, for a low concentration, when differently charged particles collide, $D_\perp$ increases as the concentration increases. For high concentration, when the EF system consists primarily of weakly charge microparticles [1] with low velocities the self-diffusion coefficient decreases sharply.

To make an engineering estimate of the dependence $D_\perp(n)$ we shall estimate the self-diffusion coefficient by the quantity $D_\perp \simeq \overline{\Delta l}^2/4\bar{\tau}$, where $\overline{\Delta l} = \tilde{V}\overline{\sin\varphi\cos\varphi}\,\tau_r$ is the transverse mean free path of a test particle; $\bar{\tau} = 1/(\beta\tilde{n}\tilde{V})$ is the average time of free flight; and, $\tilde{V}$ is the relative velocity of the scatterers with concentration $\tilde{n}$. Using the expression (3) and the fact that the impact parameter b is distributed as $2b/(2r)^2$ it is easy to obtain $\overline{\sin\varphi\cos\varphi} = \pi/8$. As a result we have

$$D_\perp = \left(\frac{\pi}{16}\right)^2 \tilde{V}^3 \tau_r^2 \beta \tilde{n}. \tag{6}$$

For a low concentration of microparticles we can set $\tilde{V} = 2V_{qM}$, $V_{qM} = V(q_M)$, $\tilde{n} = n/2$. Then

$$D_{\perp}^{*} = \left(\frac{\pi}{8}\right)^2 V_{q_M}^3 \tau_r^2 \beta n. \tag{7}$$

For a high concentration of microparticles $D_{\perp}$ must be estimated taking into account charge neutralization in collisions. Since in a collision of a test particle with charge q_M mainly with weakly charged particles approximately half the charge is lost in each collision, over M collisions occurring over the interelectrode distance d the charge of the test particle will be equal to $q_M/2^M$. For $M = d/l = d\beta\tilde{n}$, l is the mean free path, we obtain the average charge of the test particle:

$$\bar{q} = \frac{q_M}{M} \sum_{k=0}^{M} 1/2^k \simeq 2q_M/(d\beta\tilde{n}). \tag{8}$$

For a high concentration of microparticles we can set $\tilde{n} = n$, $\tilde{V} = V_q \simeq (\bar{q}E/s)2V_{q_M}/(d\beta\tilde{n})$. Substituting these quantities into Eq. (6), we obtain an estimate of the self-diffusion coefficient with high concentration

$$D_{\perp}^{**} = \frac{1}{2}\left(\frac{\pi}{4}\right)^2 V_{q_M}^3 \tau_r^2 d^{-3} \beta^{-2} n^{-2}. \tag{9}$$

Thus for a high concentration of particles the self-diffusion coefficient is inversely proportional to the squared concentration of the EF particles and decreases as this concentration increases. These estimates are in quite good agreement with the results of the numerical experiment (curves 3 and 4). Based on this, the self-diffusion coefficient can be approximated by the dependence $1/D_{\perp} = 1/D_{\perp}^{*} + 1/D_{\perp}^{**}$. Taking into account (7) and (9), we find that the maximum of $D_{\perp}$ occurs at $n = 1/(d\beta) = n_{\text{init}}$. We shall study the conditions under which the EF system is stable relative to transverse nonuniformity of the microparticle concentration. According to Eq. (2), the direction of the diffusion flux is determined by the sign of the derivative $\partial(D_{\perp}n)/\partial n$. In the case $\partial(D_{\perp}n)/\partial n > 0$ the EF system is stable, i.e., when a transverse gradient of the concentration arises the diffusion flux is directed in the direction of smaller values of n and, therefore, it equalizes the concentration in the EF system. In the case $\partial(D_{\perp}n)/\partial n < 0$ the EF system is unstable, since the concentration gradient increases. Figure 1 shows the dependence of $D_{\perp}n$ on the concentration of the fluidized microparticles. Just like the self-diffusion coefficient, the product $D_{\perp}n$ has a maximum. The concentration at which this occurs approximately corresponds to the critical concentration n_{cr} at which maximum current density and potential energy of the EF system are observed. In accordance with the adopted approximation $D_{\perp}n$ is maximum for $n = 4^{1/3}/(d\beta) = 1.6\, n_{\text{init}}$. It is obvious that for $n > n_{\text{cr}}$ the EF system is unstable, since any transverse nonuniformity of the concentration will lead to unbounded growth of the concentration. Thus in this case there occurs a unique electrodynamic collapse. It can be concluded that when the concentration of EF particles exceeds the critical concentration, the EF system is absolutely transversely unstable. It was shown previously in [1] that the EF system for $n > n_{\text{cr}}$ is also unstable in the longitudinal direction with formation of a layer of deposits on the electrode. Thus for a concentration of fluidized microparticles exceeding the critical value interparticle collisions make the EF system unstable both transverse and longitudinal to the electric field.

For $n < n_{\text{cr}}$, however, the EF system can be unstable. It should first be noted that the self-diffusion coefficient depends strongly on the particle size. For low concentration $D_{\perp}^{*} \sim r^9$. This means that as the size of the microparticles decreases the capability of the EF system to equalize its concentration owing to self-diffusion drops sharply. To estimate the effect of the nonuniformity of the electric field, associated, for example, with the out-of-parallelism of the electrodes, we shall determine the particle flux directed in the direction of decreasing field strength. According to [2], it is equal to $J_{\Delta E} = nV_{q_M}^2\tau_r(\psi/d)$, where ψ is the angle of divergence of the electrodes. Equating to this flux the diffusion flux, according to Eqs. (2) and (7), we obtain

$$\Delta d \simeq \psi\Delta x = 2\left(\frac{\pi}{8}\right)^2 V_{q_M}\tau_r \beta d \Delta n. \tag{10}$$

Changing the interelectrode separation by more than Δd results in transverse instability of the EF system. Just like the self-diffusion coefficient, the quantity Δd depends strongly on the microparticle size $\Delta d \sim r^5$ and is very small for micron-size particles. Thus for $r = 2\cdot10^{-6}$ m and $\Delta n \simeq n = 10^{11}$ m^{-3} we obtain from Eq. (10) $\Delta d = 2.5\cdot10^{-5}$ m. Thus for micron-size particles Δd could be equal in order of magnitude to the thickness of the layer of deposits which appears on the surface of the electrode as the concentration of the EF particles increases. Taking into account the direction of the flux, it can be concluded that the layer of deposits appears in the parts of the interelectrode space where the interelectrode distance is greatest. In turn the layer of deposits can result in compensation of the initial nonuniformity. Therefore the EF system can have a stationary nonuniform structure in the form of regions with different concentration of the fluidized microparticles in the interelectrode space. In the general case the dynamics of the development of the nonuniformity in the EF system, taking into account both the transverse and longitudinal instability, requires a special analysis in which the nonstationary diffusion equation is solved. However, based on the estimate made, namely, of the transverse instability of the EF system at high concentrations it is possible to explain the formation of nonuniform regions and the existence of jet flows, which were noted in [2], under conditions of electrodynamic fluidization of fine powders.

NOTATION

Here r is the radius of a microparticle; ρ is the density of a microparticle; E is the electric field strength; q_M is the maximum charge of a microparticle; $\beta = 4\pi r^2$ is the scattering cross section; m is the mass of a microparticle; g is the acceleration of gravity; n is the concentration; d is the interelectrode distance; and, s is the resistance of the medium per unit velocity of a microparticle.

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FLUIDIZATION OF MICROPARTICLES IN ELECTRIC FIELD

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The results of statistical modeling of the electrodynamic fluidization of microparticles in an electric field are outlined. The dependence of the current density and charge distribution function of the microparticles on the microparticle concentration is discussed. The limiting attainable microparticle concentration in the interelectrode space is considered. The dependence of the concentration of fluidized microparticles on their bulk concentration is considered.

Electrodynamic fluidization (EDF) of conducting powders, in which microparticles of the material move in a sufficiently strong electric field, is currently of interest as a method of fluidizing disperse materials with the aim of intensifying technological processes. On account of recharging at the electrodes, the particles perform oscillations in the interelectrode space.

In the experimental investigation of this process, it is established, in particular, that there is some limiting concentration of microparticles involved in fluidization. This limiting concentration depends on the size of the microparticles, but does not depend on the magnitude of the electric field [1]. The current density of the EDF system tends to saturation with increase in microparticle concentration.

In [2], the existence of a limiting concentration was explained by the influence of gravitational forces, which lead to asymmetry of the particle distribution in the interelectrode space limiting the EDF-particle concentration. This mechanism applies for large particles (radius $r = 100\text{-}500 \cdot 10^{-6}$ m).

In [3], the existence of a recombination mechanism limiting the EDF-particle concentration as a result of microparticle collisions was noted. However, only a few estimates were made.

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